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# Statistical production and binding energy of hypernuclei

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In nuclear reactions of high energy one can simultaneously produce a lot of hypernuclei after the capture of hyperons by nuclear residues. We consider statistical disintegration of such hypernuclear systems and the connection of fragment production with the binding energies of hyperons. It is demonstrated that the hyperon binding energies can be effectively evaluated from the yields of different isotopes of hypernuclei. The double ratio method is suggested for this purpose. The advantage of this procedure is its universality and the possibilty to involve many different isotopes. This method can also be applied for multi-strange nuclei, which binding energies were very difficult to measure in previous hypernuclear experiments.

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Образование и энергия связи гиперядер: статистический подход

В ядерных реакциях при высоких энергиях можно одновременно получить много гиперядер в результате захвата гиперонов ядерными остатками. Рассматривается статистическая дезинтеграция таких гиперядерных систем и связь образования фрагментов с энергиями связи гиперонов. Показано, что энергию связи гиперонов можно эффективно оценить по выходам различных изотопов гиперядер. Для этой цели предлагается метод двойного отношения. Преимуществом этой процедуры является ее универсальность и возможность задействовать много различных изотопов. Этот метод также может быть применен для ядер с большими значениями странности, энергии связи которых было очень трудно измерить в предшествующих гиперядерных экспериментах.

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## I. INTRODUCTION

A promising way to produce hypernuclei is to use the copious production of hyperons ($\Lambda, \Sigma, \Xi, \Omega$) in relativistic nuclear reactions with their subsequent capture by nuclei. Hypernuclei live significantly longer than the typical reaction times. Baryons with strangeness embedded in the nuclear environment allow for approaching the many-body aspect of the strong three-flavor interaction (i.e., including $u$, $d$, and $s$ quarks) at low energies. Also hypernuclei can serve as a tool to study the hyperon–nucleon and hyperon–hyperon interactions. The investigation of reactions leading to hypernuclei and the structure of hypernuclei is the progressing field of nuclear physics, since it provides complementary methods to improve traditional nuclear studies and open new horizons for studying particle physics and nuclear astrophysics (see, e.g., [1–6] and references therein).

A traditional way for the hypernuclear physics is focusing on spectroscopic information and is dominated by a quite limited set of lepton and hadron induced reactions [1, 2]. In these reactions the hyperons produced in the first interaction are directly captured by nuclei in their ground and low excited states. Kaons are often used for tagging this production. Within this method it is possible to obtain rather precisely the binding energies of hyperons inside nuclei by measuring few products of the reaction exactly. However, there are severe limitations on such methods, since the targets should be mainly stable (not radioactive). Therefore. many hypernuclear isotopes are not reachable experimentally in this case.

We emphasize a possibility to form hypernuclei in the deep-inelastic reactions leading to fragmentation processes, as they were discovered long ago [7]. One can form hypernuclei of all sizes and isospin content when these hyperons are captured by nucleons and nuclear fragments produced in same reaction events. Many experimental collaborations STAR at RHIC [8], ALICE at LHC [9], PANDA [10], CBM [11], HypHI, Super-FRS, R3B at FAIR [12, 13], BM@N, MPD at NICA [14]) plan to investigate hypernuclei and their properties in reactions induced by relativistic hadrons and ions. The limits in isospin space, particle unstable states, multiple strange nuclei and precision lifetime measurements are unique topics of these fragmentation reactions. A capture of hyperons by large nuclear residues is specially interesting since it provides a natural way to study large bulbs of hypermatter and its evolution, for example, the liquid-gas type phase transition. It was theoretically demonstrated [5, 15–21] that in such a way it is possible to produce all kind of hypernuclei including multi-strange ones. There were also experimental confirmations of such processes leading at least to single hypernuclei [12, 22, 23].

The possibility to obtain many various hypernuclei in the same reaction opens new opportunities for their investigation in comparison with previous methods. Complex multi-hypernuclear systems incorporating more than two hyperons can be created in the energetic nucleus-nucleus collisions [16, 19]. This may be the only conceivable method to go beyond double hypernuclei, and obtain new experimental information on properties of multi-hyperon systems. In this paper we demonstrate how the important knowledge on the hyperon binding energies, including in multi-strange nuclei, can be extracted from analysis of the relative yields of hypernuclei.

## II. STATISTICAL PRODUCTION OF HYPERNUCLEI FROM EXCITED HYPERNUCLEAR SYSTEMS

The hyperons are abundantly produced in high energy particle reactions, e.g., nucleus-nucleus, hadron-nucleus and lepton-nucleus collisions. The production of strangeness correlates with particle production and, usually, emission of many nucleons can accompany the production of hyperons. An initial nucleus can loose many nucleons, and, as known from normal nucleus interactions, these processes can lead to a high excitations of remaining residual nuclei, see e.g. [24–26]. Therefore, the capture of a produced hyperon will be also realized mostly at the excited nuclei. As a result such deep-inelastic processes can form large hyper-residues with very broad distribution in mass and excitation energy. As was demonstrated in our previous works [17–19], the yields of the hypernuclear residues in peripheral ion collisions will saturate with energies above 3–5 A GeV (in the laboratory frame).

The reactions of formation of excited nuclear residues in high-energy nucleus-nucleus and hadron-nucleus collisions were intensively studied in connection with fragmentation and multifragmentation processes. In particular, masses and excitation energies of the residues are known from experimental and theoretical works, e.g., Refs. [19, 25]. At high excitation energy the dominating decay mode is a multifragmentation process [24, 27, 28]. The hyperon interactions in a nucleus are similar to nuclear ones, and its potential is around 2/3 of the nucleon one. Therefore, we believe, that an addition of few hyperons to a multi-nucleon system can not change its disintegration behavior. According to the present understanding, multifragmentation is a relatively fast process, with a characteristic time around 100 fm/c, where, nevertheless, a high degree of equilibration (chemical equilibrium) is reached. This is a consequence of the strong interaction between baryons, which are in the vicinity of each other in the freeze-out volume.

The statistical models have demonstrated very good agreement in comparison with fragmentation and multifragmentation data [24, 25, 27, 29]. It is naturally to extend the statistical approach for hypernuclear systems. It is also important that the same numerical methods used previously for execution of the models can be extended. The statistical multifragmentation model (SMM), which was very successfully applied for description of normal multifragmentation processes, was generalized for hypernuclei in Ref. [15]. A transition from the compound hyper-nucleus to the multifragmentation regime was also under investigation [5, 15]. The break-up channels are generated according to their statistical weight. The Grand Canonical approximations leads to the following average yields of individual fragments with the mass (baryon) number $A$, charge $Z$, and the $\Lambda$-hyperon number $H$:

$$Y_{\mathrm{A,Z,H}} = g_{\mathrm{A,Z,H}} \cdot V_f \frac{A^{3/2}}{\lambda_T^3} \exp\left[-\frac{1}{T}\left(F_{A,Z,H} - \mu_{AZH}\right)\right], \qquad \mu_{AZH} = A\mu + Z\nu + H\xi. \tag{1}$$

Here $T$ is the temperature, $F_{A,Z,H}$ is the internal free energies of these fragments, $V_f$ is the free volume available for the translation motion of the fragments, $g_{\mathrm{A,Z,H}}$ is the spin degeneracy factor of species $(A, Z, H)$, $\lambda_T = (2\pi\hbar^2/m_N T)^{1/2}$ is the baryon thermal wavelength, $m_N$ is the average baryon mass. The chemical potentials $\mu$, $\nu$, and $\xi$ are responsible for the mass (baryon) number, charge, and strangeness conservation in the system, and they can be numerically found from the corresponding conservation laws. In this model the statistical ensemble includes all break-up channels composed of baryons and excited fragments. The primary fragments are formed in the freeze-out volume $V$. We use the excluded volume approximation $V = V_0 + V_f$, where $V_0 = A_0/\rho_0$ ($A_0$ is the total baryon number and $\rho_0 \approx 0.15$ fm$^{-3}$ is the normal nuclear density), and parametrize the free volume $V_f = \kappa V_0$, with $\kappa \approx 2$, as taken in description of experiments in Refs. [25, 27, 29].

The following model development depends on the physical processes which are the most adequate to the analyzed reactions. For example, nuclear clusters in the freeze-out volume can be described in the liquid-drop approximation: Light fragments with mass number $A < 4$ are treated as elementary particles with corresponding spins and translational degrees of freedom ("nuclear gas"). Their binding energies were taken from experimental data [1, 2, 24]. The fragments with $A = 4$ are also treated as gas particles with table masses, however, some excitation energy is allowed $E_x = AT^2/\varepsilon_0$ ($\varepsilon_0 \approx 16$ MeV is the inverse volume level density parameter [24]), that reflects a presence of excited states in $^4$He, $^4_\Lambda$H, and $^4_\Lambda$He nuclei. Fragments with $A > 4$ are treated as heated liquid drops. In this way one can study the nuclear liquid-gas coexistence of hypermatter in the freeze-out volume. The internal free

energies of these fragments are parametrized as the sum of the bulk ($F_A^B$), the surface ($F_A^S$), the symmetry ($F_{AZH}^{\rm sym}$), the Coulomb ($F_{AZ}^C$), and the hyper energy ($F_{AH}^{\rm hyp}$):

$$F_{A,Z,H} = F_A^B + F_A^S + F_{AZH}^{\rm sym} + F_{AZ}^C + F_{AH}^{\rm hyp} \quad . \tag{2}$$

Here, the first three terms are written in the standard liquid-drop form [24]:

$$F_A^B = \left(-w_0 - \frac{T^2}{\varepsilon_0}\right) A \quad , \tag{3}$$

$$F_A^S = \beta_0 \left(\frac{T_c^2 - T^2}{T_c^2 + T^2}\right)^{5/4} A^{2/3} \quad , \tag{4}$$

$$F_{AZH}^{\rm sym} = \gamma \frac{(A-H-2Z)^2}{A-H} \quad , \tag{5}$$

where $w_0 = 16$ MeV, $\beta_0 = 18$ MeV, $T_c = 18$ MeV and $\gamma = 25$ MeV are the model parameters which are extracted from nuclear phenomenology and provide a good description of multifragmentation data [24, 25, 27, 29]. The Coulomb interaction of fragments is described within the Wigner-Seitz approximation, and $F_{AZ}^C$ is taken as in the ref. [15, 24]:

$$F_{AZ}^C(V) = \frac{3}{5}\left[1 - \left(\frac{V_0}{V}\right)^{1/3}\right] \frac{(eZ)^2}{r_0 A^{1/3}} \quad . \tag{6}$$

where $r_0 = 1.2$ fm and $e$ denotes the electron charge.

For our purpose the free hyper-energy term $F_{AH}^{\rm hyp}$ is very important. We assume that it is determined only by the binding energy of hyper-fragments. Presently, only few ten masses of single hypernuclei (mostly light ones) are experimentally established [1, 2], and, practically, there are no reliable measurements of double hypernuclei. There are only few theoretical estimations of their masses based on a limited description of the available data. In Ref. [15] we have suggested a liquid drop hyper term:

$$F_{AH}^{\rm hyp} = (H/A) \cdot (-10.68A + 21.27A^{2/3}) MeV. \tag{7}$$

In this formula the binding energy is proportional to the share of hyperons in matter ($H/A$). The second part is the volume contribution minus the surface one: It is a normal liquid-drop parametrization assuming saturation of the nuclear interaction. The linear dependence at small $H/A$ is in agreement with theoretical predictions [30] for hyper matter. As was demonstrated in Refs. [5, 15] this parametrization of the hyperon binding energy describes available experimental data quite well. It is important that two boundary physical effects are correctly reproduced: The binding energies of light hypernuclei (if a hyperon substitutes

a neutron) can be lower than in normal nuclei, since the hyperon-nucleon potential is smaller than the nucleon-nucleon one. However, since the hyperon can take the lowest s-state, it can increase the nuclear binding energies, specially for large nuclei.

## III. METHODS OF THE YIELD DOUBLE RATIOS FOR SEPARATION OF THE HYPERON BINDING

We provide various methods demonstrating how the hypernuclei yields can be used for addressing the hyperon binding energies. The methods use different assumptions on fragments in the freeze-out, however, the same physical idea is realized. In applications it would be reasonable to select one of the methods by taking into account the particular experimental opportunities to measure some hypernuclear fragments, and by identifying the processes of their production.

### A. Liquid-drop approximation for fragments

It is convenient to rewrite the above formulas in order to show separately the binding energy $E_A^{\rm bh}$ of one hyperon at the temperature $T$ inside a hypernucleus with $A, Z, H$ :

$$E_A^{\rm bh} = F_{A,Z,H} - F_{A-1,Z,H-1} \ . \tag{8}$$

Since $\Lambda$-hyperon is usually bound, this value is negative. Then the yield of hypernuclei with an additional $\Lambda$ hyperon can be recursively written by using the former yields:

$$Y_{\rm A,Z,H} = Y_{\rm A-1,Z,H-1} \cdot C_{\rm A,Z,H} \cdot \exp\left[-\frac{1}{T}\left(E_A^{\rm bh} - \mu - \xi\right)\right], \tag{9}$$

where $C_{\rm A,Z,H} = (g_{\rm A,Z,H}/g_{\rm A-1,Z,H-1}) \cdot (A^{3/2}/(A-1)^{3/2})$ depends mainly on the ratio of the spin factors of $A, Z, H$ and $A-1, Z, H-1$ nuclei, and very weakly (especially for large nuclei) on $A$. Since in the liquid-drop approximation we assume that the fragments with $A > 4$ are excited and do populate many states (above the ground) according to the given temperature dependence of the free energy, then we take $g_{\rm A,Z,H} = 1$. Within SMM we can connect the relative yields of hypernuclei with the hyperon binding energies. It is interesting that in this formulation one can use other parametrizations to describe nuclei in the freeze-out. This statistical approach is quite universal, and only small corrections, like the table-known spins and energies, may be required for more extensive consideration.

We suggest the following receipt for obtaining information on the binding energies of hyperons inside nuclei. Let us take two hyper-nuclei with different masses, $(A_1, Z_1, H)$ and

$(A_2, Z_2, H)$, together with nuclei which differ from them only by one $\Lambda$ hyperon. We consider the double ratio ($DR$) of $Y_{\mathrm{A_1,Z_1,H}}/Y_{\mathrm{A_1-1,Z_1,H-1}}$ to $Y_{\mathrm{A_2,Z_2,H}}/Y_{\mathrm{A_2-1,Z_2,H-1}}$. We obtain from the above formulae

$$DR_{A_1A_2} = \frac{Y_{\mathrm{A_1,Z_1,H}}/Y_{\mathrm{A_1-1,Z_1,H-1}}}{Y_{\mathrm{A_2,Z_2,H}}/Y_{\mathrm{A_2-1,Z_2,H-1}}} = \alpha_{A_1A_2}\exp\left[-\frac{1}{T}\left(\Delta E^{\mathrm{bh}}_{A_1A_2}\right)\right], \tag{10}$$

where

$$\Delta E^{\mathrm{bh}}_{A_1A_2} = E^{\mathrm{bh}}_{A_1} - E^{\mathrm{bh}}_{A_2}, \tag{11}$$

and the ratio of the $C$-coefficients we denote as

$$\alpha_{A_1A_2} = C_{\mathrm{A_1,Z_1,H}}/C_{\mathrm{A_2,Z_2,H}}\ . \tag{12}$$

### B. Fragments with fixed binding energies

We note that the same $DR$ expression may be deduced in a standard statistical way (see, e.g., [31]), without sophisticated description of the hot fragments in the freeze-out volume. We consider only fixed fragment binding energies without a temperature dependence. This physical condition can be adequate for lightest fragments. In this case after disintegration of nuclear systems the grand-canonical yields of a normal nucleus in the ground state can be written as

$$Y_{\mathrm{A,Z}} = g_{\mathrm{A,Z}}\cdot V_f\frac{A^{3/2}}{\lambda_T^3}\exp\left[-\frac{1}{T}\left(E^b_{A,Z} - \mu_{A,Z}\right)\right],$$
$$\mu_{A,Z} = A\mu + Z\nu, \tag{13}$$

where $A$ is the nucleon number, $g_{\mathrm{A,Z}}$ is the standard spin factor, and $E^b_{A,Z}$ is the nucleus ground state binding energy. If the system contains one or few hyperons this formula can be easily generalized for producing hypernuclei with additionally captured $H_c \geq 1$ hyperons as (compare with eq.(1))

$$Y_{\mathrm{A',Z,H_c}} = g_{\mathrm{A',Z,H_c}}\cdot V_f\frac{A'^{3/2}}{\lambda_T^3}\exp\left[-\frac{1}{T}\left(E^b_{A',Z,H_c} - \mu_{A,Z,H_c}\right)\right],$$
$$\mu_{A,Z,H_c} = A'\mu + Z\nu + H_c\xi, \tag{14}$$

where $A' = A + H_c$, and $g_{\mathrm{A',Z,H_c}}$ is a new spin factor, which should only slightly differ from $g_{\mathrm{A,Z}}$, since a hyperon is usually located in s-state. $E^b_{A,Z,H_c}$ differs from the normal binding energy only via the pure hyperon binding energy $E^{bh}_{A',Z,H_c}$:

$$E^b_{A',Z,H_c} = E^b_{A,Z} + E^{bh}_{A',Z,H_c}, \tag{15}$$

which leads to the equivalence with (8) for cold fragments. Therefore, the corresponding double ratio $DR_{A_1^{'}A_2^{'}}$ of the yields of hypernuclei with mass numbers $A_1^{'} = (A_1 + H_c)$ and $A_2^{'} = (A_2 + H_c)$ can be again written as eq,(10), however, with

$$\Delta E^{\rm bh}_{A_1^{'}A_2^{'}} = E^{bh}_{A_1^{'},Z_1,H_c} - E^{bh}_{A_2^{'},Z_2,H_c}, \tag{16}$$

and with the coefficient

$$\alpha_{A_1^{'}A_2^{'}} = \frac{g_{\rm A_1^{'},Z_1,H_c}/g_{\rm A_1^{'}-1,Z_1,H_c-1}}{g_{\rm A_2^{'},Z_2,H_c}/g_{\rm A_2^{'}-1,Z_2,H_c-1}} \cdot \frac{A_1^{'3/2}/(A_1^{'}-1)^{3/2}}{A_2^{'3/2}/(A_2^{'}-1)^{3/2}}. \tag{17}$$

In this case, as was mentioned, the hypernuclei are considered in the final states (cold ones) in the freeze-out.

As one can simply deduce from eq.(10), the logarithm of the double ratio is directly proportional to the difference of the hyperon binding energies in $A_1$ and $A_2$ hypernuclei, $\Delta E^{\rm bh}_{A_1A_2}$, divided by temperature. Therefore, we can finally rewrite the relation between the hypernuclei yields and the hyperon binding energies as

$$\Delta E^{\rm bh}_{A_1A_2} = T \cdot \left[ ln(\alpha_{A_1A_2}) - ln(DR_{A_1A_2}) \right]. \tag{18}$$

In some cases we expect a large difference in hyperon binding energy in both nuclei. For example, according to the liquid-drop approach (see eq. (2)) it can be when the difference between $A_1$ and $A_2$ is essential (i.e., the mass number $A_2$ is much larger than $A_1$). The influence of the pre-exponential $\alpha$ coefficients is small and can be directly evaluated, depending on the selected hypernuclei. This opens a possibility for the explicit determination of the binding energy difference from experiments. In this case, it is necessary to measure some number of the hypernuclei in one reaction and select the corresponding pairs of hypernuclei. One has to identify such hypernuclei, for example, by the correlations and vertex technique. However, there is no need to measure very precisely the momenta of all particles produced in the reaction (including after the week decay of hypernuclei) to obtain their binding energy, as it must be done in processes of direct capture of hyperons in the ground and slightly excited states of the target nuclei (e.g., in missing mass experiments [2, 32]). Therefore, our procedure perfectly suits for investigation of hypernuclei in the high-energy deep-inelastic hadron and ion induced reactions.

### C. Isobar double ratios

Another interesting way for this study is to use the double ratios of yields with the same mass numbers for light and heavy pairs. This case is easy to illustrate for cold fragments. The so-called strangeness population factor $S$ was introduced in Ref. [33] for interpretation of light hypernuclei production in relativistic heavy-ion collision (at momenta of 11.5 A GeV/c):

$$S = \frac{Y_{^3\mathrm{H}_\Lambda}/Y_{^3\mathrm{He}}}{Y_\Lambda/Y_\mathrm{P}} \tag{19}$$

Generally, if we involve the pairs of nuclei which differ by one proton instead of Λ-hyperon, we can write the isobar double ratio:

$$DR^I_{A_1A_2} = \frac{Y_{\mathrm{A}_1,\mathrm{Z}_1,\mathrm{H}}/Y_{\mathrm{A}_1,\mathrm{Z}_1+1,\mathrm{H}-1}}{Y_{\mathrm{A}_2,\mathrm{Z}_2,\mathrm{H}}/Y_{\mathrm{A}_2,\mathrm{Z}_2+1,\mathrm{H}-1}} = \alpha^I_{A_1A_2}\exp\left[-\frac{1}{T}\left(\Delta E^\mathrm{b}_X\right)\right], \tag{20}$$

where

$$\alpha^I_{A_1A_2} = \frac{g_{\mathrm{A}_1,\mathrm{Z}_1,\mathrm{H}}/g_{\mathrm{A}_1,\mathrm{Z}_1+1,\mathrm{H}-1}}{g_{\mathrm{A}_2,\mathrm{Z}_2,\mathrm{H}}/g_{\mathrm{A}_2,\mathrm{Z}_2+1,\mathrm{H}-1}}, \tag{21}$$

and the binding energy difference between 4 fragments

$$\Delta E^\mathrm{b}_X = (E^b_{A_1,Z_1,H} - E^b_{A_2,Z_2,H}) - (E^b_{A_1,Z_1+1,H-1} - E^b_{A_2,Z_2+1,H-1}). \tag{22}$$

The last expression (22) can not be factorized into the binding energies of normal nuclei with $A_1$ and $A_2$ and the part related only to the hyperon binding (as it was possible in formulae (8) and (15)), since it includes also the difference of the hyperon binding in hyper-nuclei with $Z+1$. Therefore, it requires complicated calculations of the coupled equations for extracting the hyperon binding. In addition, extra experimental isobar measurements will be necessary. Still, the convenient application of $DR^I$ can be found for single hypernuclei with $H=1$, when for the pair nuclei (at $H-1=0$ and $Z+1$) there are only normal nuclei with known binding energies. In this case one can rewrite the formula (18) as

$$\Delta E^\mathrm{bh}_{A_1A_2} = T\cdot\left[ln(\alpha^I_{A_1A_2}) - ln(DR^I_{A_1A_2})\right] + \Delta E^\mathrm{GS}_{A_1A_2}, \tag{23}$$

where $\Delta E^\mathrm{GS}_{A_1A_2}$ is the difference of the ground state binding energies of non-strange nuclei:

$$\Delta E^\mathrm{GS}_{A_1A_2} = (E^b_{A_1,Z_1+1} - E^b_{A_2,Z_2+1}) - (E^b_{A_1-1,Z_1} - E^b_{A_2-1,Z_2}). \tag{24}$$

In the above mentioned example, as was obtained by AGS-E864 collaboration [33], $S = 0.36$ (with large error bars $+-0.26$) for the most central collisions and for fragments produced in the midrapidity region. Since the binding energies of all nuclei in $S$-factor (19) are known from other experiments we can from formula (20) evaluate the temperature of the

excited hyper-source leading to producing of these fragments and hypernuclei: The found chemical temperature is around $T \approx 5.5$ MeV. This is typical for the nuclear liquid-gas phase coexistence region under condition that all available baryons are produced in a dynamical way. It is also consistent with the limited equilibration of fragments reported previously for central heavy ion collisions [34].

### D. Novel directions for application of the method

It is clear from the subsections **A** and **B** that the suggested double ratio approach can be applied to hypernuclei with any number of hyperons: As obvious, the equations (1) and (10) can be used for $H > 1$. One can reach a multi-strange residues in nuclear reactions with a quite large probability [19], and a very wide mass/isospin range will be available for examination. As a result, one can get direct experimental evidences for hyperon binding energies in double/triple hypernuclei and on influence of the isospin on hyperon interactions in multi-hyperon nuclear matter.

The connection between the relative hyperon binding energies $\Delta E^{\rm bh}_{A_1A_2}$ and its absolute values can be done straightforward: It will be sufficient to make normalization to the binding energy of a known hypernuclei (e.g., $A_2$) obtained with other methods. However, even relative values are extremely important, when we pursue a goal to investigate the trends of the hyperon interaction in different nuclear surroundings, e.g., neutron-rich or neutron-poor ones. Novel conclusions can be obtained by comparing yields of neutron-rich and neutron-poor hypernuclei. The isospin influence on the hyperon interaction in matter (revealing in the hyperon binding energies) will be possible to extract directly in experiment by using the formula (18). Especially multi-strange nuclear systems would be interesting, since they can give info on evolution of the hyperon-hyperon interaction depending on strangeness. These measurements are important for many astrophysical sites, for example, for understanding the neutron star structure [35, 36].

The comprehensive analysis of double hypernuclei and multi-hyperon nuclei is possible within this approach, and it seems the only realistic way to address experimentally the hyperon binding in multi-strange nuclei. This is an important advantage over the standard hypernuclear measurements. Actually, the disintegration of hot hyper-residues suits in the best for this examination since all kind of normal and hyper-fragments can be formed within the same statistical process. As was previously established in multifragmentation studies, the selection of adequate reaction conditions can be experimentally verified.

## IV. INFLUENCE OF PROCESSES ACCOMPANYING THE STATISTICAL FRAGMENTATION

The conception of the statistical formation of fragments in the freeze-out volume suggests, first, the existence of some important parameters (e.g., temperature), and, second, it may suggest the phenomena (e.g., secondary interactions) which can finally change the baryon composition of fragments after they leave the freeze-out. All these effects were under careful examination previously in multifragmentation processes. We outline how it could be taken into account in the hypernuclear case.

### A. Temperature and the freeze-out state

In order to find $\Delta E^{\rm bh}_{A_1A_2}$ in experiment within the double ratio approach, we should determine the temperature $T$ of the disintegrating hypernuclear system. This observable was also under intensive investigation recent years in connection with multi-fragment formation. There were suggested various methods: using kinetic energies of fragments, excited states population, and isotope thermometers [28, 37, 38]. Usually, all evaluations give the temperature around 4–6 MeV in the very broad range of the excitation energies (at $E^* > 2-3$ MeV per nucleon), providing so-called a plateau-like behavior of the caloric curve [24, 28]. The isotope thermometer method is the most promising, since it allows for involving a large number of normal measured isotopes in the same reactions which produce hypernuclei. The corresponding experimental and theoretical research were performed last years to investigate better the temperature and isospin dependence of the nuclear liquid-gas type phase transition [38–41]. We believe that the great experience accumulated previously in this field gives a chance to find a reliable temperature of the hypernuclear residues.

In this case it would be instructive to select the reaction conditions leading to similar freeze-out states. The freeze-out restoration methods were extensively tested previously: In particular, the masses and excitation energies of the hypernuclear residues can be found with a sufficient precision [42, 43]. One can analyze the subsequent ranges of the excitation energy (from low to very high ones) to investigate the evolution of the hypernuclei with the temperature and the phase transition in hyper-matter. It is specially interesting to move into the neutron-rich domain of the nuclear chart, by selecting neutron-rich target or projectiles, in addition to sorting out the various excitations of the sources.

### B. Secondary de-excitation corrections

We may expect that the primary fragments and hyper-fragments (specially, large ones) in the freeze-out volume could be excited, therefore, they should fastly decay after escaping the freeze-out. For low excited sources the fragment excitation energy should roughly correspond to the compound nucleus temperature. As was established in theory and multifragmentation experiments [44], the internal fragment excitations are around 2–3 MeV per nucleon for highly excited residue sources. The secondary de-excitation influences all 4 fragments entering the double ratio and the fragments should loose few nucleons. The investigation of similar nuclear decay processes of excited nuclei in normal multifragmentation reactions tell us that if the difference in mass between initial fragments is small then the mass difference between final products will be small too. Following this de-excitation the mass numbers will change and we expect a smooth transformation of $\Delta E^{\rm bh}_{A_1A_2}$ versus the variation of mass difference $\Delta A = (A_2 - A_1)$: The new yields and mass numbers should be used for the final estimate. This effect can be investigated in the framework of the evaporation model for large ($A \gtrsim 16$) hypernuclei developed in Ref. [45]. There was demonstrated that mostly neutrons and other light normal particles will be emitted from hot large hyper-fragments, since the hyperons have a larger binding energy. Such an effect should not change dramatically the general form of the $\Delta E^{\rm bh}_{A_1A_2}$ dependence on $\Delta A$. For small ($A \lesssim 16$) hot hyper-fragments formed in the freeze-out volume the most adequate model is the Fermi-break-up [24], similar to the one for normal fragments. It was generalized for hypernuclei in Ref. [46]. The consequences of these secondary decay processes will be the subject of our future studies.

## V. CONCLUSION

During last six decades there is permanent increasing the number of measured hypernuclei with their binding energies. However, the progress is very slow: Because of the special requirements on targets in hadron and lepton induced reactions, the traditional hypernuclear methods (e.g., the missing mass spectroscopy) can address only a small number of isotopes. Also the development of the detectors for measuring nearly all produced particles with their exact kinetic energies is very expensive and not always practical, that makes problems for a desirable acceleration of the studies.

The suggested double ratio method is related to deep inelastic reactions producing all kind of hypernuclei with sufficiently large cross-sections in the multifragmentation process.

This is a typical case for relativistic ion-ion and hadron-ion collisions. Only the identification of hypernuclei is required, and, as demonstrated in recent ion experiments, there are effective ways to perform it. The experimental extraction of the difference in the hyperon binding energies between hypernuclei ($\Delta E^{\rm bh}_{A_1A_2}$) is a novel and practical way to pursue hypernuclear studies. The advantage of this method over the traditional hypernuclear ones is that the exact determination of all produced particles parameters (with their decay products) is not necessary. Only relative measurements are necessary for this purpose, therefore, one can address similar weak-decay chains and their products. For example, if we take the pairs of large hyper isotopes, they undergo weak decay in a non-mesonic channel that can be found by products far from the collision point with the vertex technique. The correlation the produced isotopes and particles is an adequate information for the double ratios.

Even more interesting and important that with this method one can also determine the difference of hyperon binding energies in double and multi-hypernuclei. This gives an access to hyperon-hyperon interactions and properties of multi-hyperon matter. It is very difficult to measure the hyperon binding for exotic (neutron-rich and neutron-poor) nuclear species within traditional hypernuclear experiments. On the other hand, the hypernuclei with extreme isospin can be easily obtained in deep-inelastic reactions. Most of them may have the statistical origin and the suggested method opens an effective way for extension of the hypernuclear research.

We believe such kind of research would be possible at the new generation of ion accelerators of intermediate energies, as FAIR (Darmstadt), NICA (Dubna), and others. It is promising that new advanced experimental installations for the fragment detection will be available soon [47, 48].

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